KSME Journal, Vol. 3, No. 2, pp. 103~112, 1989.

FINITE CRACK PROPAGATION IN A MICROPOLAR ELASTIC SOLID

Seog Young Han*, M.N.L. Narasimhan and T.C. Kennedy*****

(Received July 3, 1989)

The dynamic propagation of a finite crack under mode-Ⅰ loading in a micropolar elastic solid is investigated. By using an integral transform method, a pair of two-dimensional singular integral equations governing stress and couple stress is formulated in terms of displacement transverse to the crack, macro and micro rotations, and microinertia. These equations are solved numerically, and solutions for dynamic stress intensity and couple stress intensity factors are obtained by utilizing the values of the strengths of the square root singularities in macrorotation and the gradient of microrotation at the crack tips. The motion of the crack tips and the load on the crack surface are not prescribed in the formulation of the problem. Therefore, the method of solution is applicable to nonuniform rates of propagation of a crack under an arbitrary time-dependent load on the crack surface. As an example, the diffraction of a micropolar dilatational wave by a stationary crack is considered. The behavior of the microrotation field and the dynamic couple stress intensity factor, influenced by microinertia, in addition to the dynamic stress intensity factor, are examined. The classical elasticity solution for the corresponding problem arises as a special case when the micropolar moduli are dropped from the present solution.

Key Words: Micropolar, Microcontinuum, Stress Intensity Factor, Couple Stress, Couple Stress Intensity Factor, Microrotation

1. INTRODUCTION

In dynamic crack propagation, a number of steady state and transient problems have been investigated from the viewpoint of classical elasticity theory. Yoffee(1951), Craggs (1960), and Sih and Loeber(1969) have investigated steady state problems. Transient problems were first considered by Broberg(1960), with subsequent contributions by Baker(1962), Freund(1972) and Thau and Lu(1971). Most of these studies have dealt with semi-infinite crack problems which are tractable. However, in real situations, cracks are of finite length; and while the investigation of the dynamic interaction of the two tips of a finite crack is very important, conventional methods of analysis, such as the Wiener-Hopf technique, cannot be easily applied to finite crack problems due to the mathematical complexities of analysis. The problem of diffraction of elastic waves by a finite crack has been discussed by Eringen and Suhubi(1974). Kim(1977) investigated the dynamic propagation of a finite crack in general terms for (1) a stationary crack, (2) a crack propagating at constant speed, and (3) a crack which suddenly stops after propagation at constant speed. All of the above studies have dealt with classical elastic media. However, to the best of our knowledge, there have been no investigations of dynamic crack propagation in micropolar elastic media.

The micropolar theory of elasticity, developed by Eringen and Suhubi(1964) and Eringen(1966), incorporates the properties of the microstructure of a material with in the continuum framework. A micropolar continuum may be regarded as a classical continuum, each point of which is assigned another continuum with an additional rotational degree of freedom besides translation. A consequence of this feature is that the medium can support couple stress, spin and microinertia. The constitutive theory for micropolar media consists of a stress constitutive equation involving microrotation and a couple stress constitutive equation involving the gradients of microrotation. The field equations are augmented by a conservation law for microinertia.

The object of this study is to obtain the dynamic stress and the dynamic couple stress intensity factors for a finite crack whose tips may propagate nonuniformly in time under an arbitrary time dependent normal load on the crack surface. By using an integral transform method, a pair of two-dimensional singular integral equations governing dynamic stress and dynamic couple stress is formulated. The present analysis demonstrates that both the macrorotation and the gradient of the microrotation of the crack surface can be determined by solving these singular inergral equations. In addition, it is shown that both the dynamic stress and the couple stress intensity factors can be obtained by utilizing the values of the strengths of the square root singularities in macrorotation and the gradient of the microrotation at the crack tips. In this analysis, we have also investigated the behavior of the microrotation field and the dynamic couple stress intensity factor, influenced by microinertia, which have no conterparts in the classical theory of elasticity. The classical elasticity solution for the dynamic finite crack propagation problem arises as a special case when the micropolar moduli are dropped from the present solution.

*Rolling Department, Research Institute of Industrial Science and Technology, Pohang 790-330, Korea.
**Professor of Department of Mathematics, Oregon State Univ.
***Associate Professor of Department of Mechanical Engineering, Oregon State Univ.

2. STATEMENT OF PROBLEM

A plane finite crack is contained in an unbounded medium as shown in Fig. 1. The body is micropolar, linearly elastic, isotropic and homogeneous, and the body force and the body couple are assumed to be negligible. A Cartesian coordinate system which has been normalized by the half crack length is introduced in such a way that the crack surface is initially defined by $-1<x<1$, $y=0_{\pm}$, $-\infty<z<\infty$. The time variable used in this study has also been normalized by the time for the micropolar dilatational wave to travel half of the crack length. As a result of this normalization, the micropolar dilatational wave speed is equal to unity. The propagation distances of the right and left crack tips are denoted by $a_{+}(t)$, and $a_{-}(t)$, respectively. Thus, the positions of the crack tips at time, t, are given by $x=\pm 1\pm a_{\pm}(t)$, $y=0_{\pm}$. The crack tip velocities $c_{\pm}(t)$ are such that $c_{\pm}(t)=0$ for $t<0$ and $0<c_{\pm}(t)<c_R$ for $t\geq 0$, where c_R is the classical Rayleigh wave speed. The field equations of micropolar linear elasticity are

$$(C_D^2+C_3^2)\,u,_{xx}+(C_D^2-C_S^2)\,v,_{xy}+(C_S^2+C_3^2)\,u,_{yy}+C_3^2\phi,_{y}=u,_{tt} \tag{1a}$$

$$(C_D^2+C_3^2)\,v,_{xx}+(C_D^2-C_S^2)\,u,_{xy}+(C_s^2+C_3^2)\,v,_{yy}+C_3^2\phi,_{x}=v,_{tt} \tag{1b}$$

and

$$C_4^2(\phi,_{xx}+\phi,_{yy})+\frac{C_3^2}{j}(v,_{x}-u,_{y})-\frac{2}{j}C_3^2\phi=\phi,_{tt} \tag{1c}$$

where u and v are the x-and y-components of the displacement vector and ϕ is z-component of the microrotation vector, and

$$C_D^2=\frac{\lambda+2\mu}{\rho},\qquad C_S^2=\frac{\mu}{\rho},$$
$$C_3^2=\frac{\kappa}{\rho},\qquad C_4^2=\frac{\gamma}{\rho j}.$$

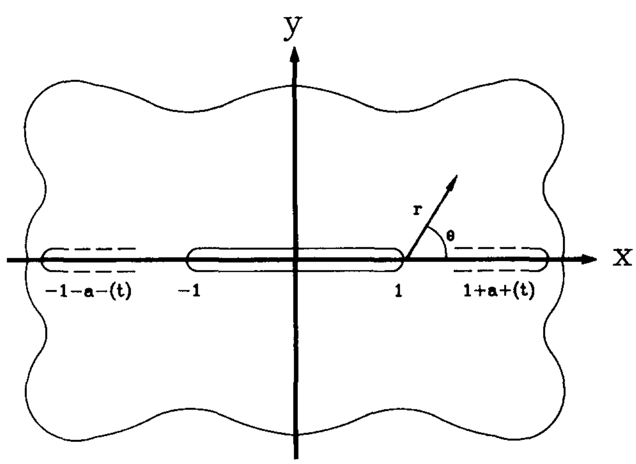

Fig. 1 Geometry of the problem

The quantities λ, μ, κ and γ are the micropolar moduli, and ρ and j are the mass density and the microinertia density, respectively, of the medium, which are treated as constants.

The general solution of this problem is the superposition of the solutions to the following: (1) the problem of the crack-free region subjected to uniaxial tension, $\sigma(x, t)$, and (2) the problem of the crack opened out by normal pressure, $\sigma(x, t)$, with no loading at infinity. Since we are interested in the dynamic stress and couple stress intensity factors, only problem (2) is considered since both of the stress intensity factors may be derived from it. By symmetry, this problem is equivalent to the problem of determination of stress distribution in the half plane $y\geq 0$, when its boundary is subjected to the following conditions:

$$\left.\begin{aligned} t_{yy}(x,0,t)&=-\sigma(x,t) \quad (2a)\\ M_{yz}(x,0,t)&=0 \quad (2b)\end{aligned}\right\}-1-a_{-}(t)<x<1+a_{+}(t),$$

$$\left.\begin{aligned} v(x,0,t)&=0 \quad (2c)\\ \phi(x,0,t)&=0 \quad (2d)\end{aligned}\right\}\ x>1+a_{+}(t)\ \text{or}\ x<-1-a_{-}(t),$$

and

$$t_{yx}(x,0,t)=0 \qquad 0\leq|x|<\infty, \tag{2e}$$

where t_{yy}, t_{yx}, σ and M_{yz} represent, respectively, the normal stress component, the shear stress component, the uniaxial tension, and pertinent couple stress component, normalized with respect to the shear modulus of the material. The initial conditions are

$$\begin{aligned} &u(x,y,0)=0,\\ &v(x,y,0)=0,\\ &\phi(x,y,0)=0,\\ &u,_{t}(x,y,0)=0,\\ &v,_{t}(x,y,0)=0,\\ &\phi,_{t}(x,y,0)=0, \end{aligned} \tag{3}$$

for all x and y.

The components of stress, displacement and microrotation must vanish as $(x^2+y^2)\rightarrow\infty$. Moreover, the symmetry with respect to the y-axis provides the additional conditions,

$$\left.\begin{aligned} v(x,0,t)&=v(-x,0,t),\\ \phi(x,0,t)&=-\phi(-x,0,t).\end{aligned}\right.\ -\infty<x<\infty, \tag{4}$$

3. FORMULATION OF INTERGRAL EQUATIONS

Since the problem is symmetric with respect to $y=0$, only the upper half space is considered and $y=0$ is used rather than $y=0_{+}$ for the y-coordinate of th crack surface. Integral transforms are employed to reduce the partial differential equations (1) to ordinary differential equations. First, the time, t, is eliminated by application of the Laplace transform,

$$\bar{f}(x,y,p)=\int_0^{\infty}f(x,y,t)e^{-pt}dt, \tag{5}$$

where the initial conditions are used. Second, Fourier trigonometric transforms are used to suppress y. They are defined by

$$\bar{\bar{f}}(x,s,p)=\int_0^{\infty}\bar{f}(x,y,p)\cos(sy)\,dy$$

$$\bar{\bar{f}}(x, s, p) = \int_0^\infty \bar{f}(x, y, p)\sin(sy)dy \tag{6}$$

The Fourier cosine transform is applied to the Laplace transform of equation (1a) and a Fourier sine transform is applied, respectively, to the Laplace transform of equations (1b) and (1c). The resulting equations are

$$\bar{\bar{u}}_{,xx} - (A^2s^2+p^2)\bar{\bar{u}} + B^2s\bar{\bar{v}}_{,x} + sC^2\bar{\bar{\phi}} = (B^2+C^2-A^2)\bar{v}_{,x} \tag{7a}$$

$$A^2\bar{\bar{v}}_{,xx} - (s^2+p^2)\bar{\bar{v}} - B^2s\bar{\bar{u}}_{,x} - C^2\bar{\bar{\phi}}_{,x} = -s\bar{v}, \tag{7b}$$

and

$$\theta^2\bar{\bar{\phi}}_{,xx} - \left(\theta^2s^2 + \frac{2\epsilon}{j} + \frac{p^2}{C_2{}^2}\right)\bar{\bar{\phi}} + \frac{\epsilon}{j}(s\bar{\bar{u}} + \bar{\bar{v}}_{,x}) = -\theta^2s\bar{\phi}, \tag{7c}$$

where

$$A^2 = \frac{C_S{}^2 + C_3{}^2}{C_M}, \quad B^2 = \frac{C_D{}^2 - C_S{}^2}{C_M}, \quad C^2 = \frac{C_3{}^2}{C_M},$$
$$\epsilon = \frac{C_3{}^2}{C_S{}^2}, \quad \theta^2 = \frac{C_4{}^2}{C_S{}^2} = \frac{\gamma}{\mu j},$$
$$C_2{}^2 = \frac{C_S{}^2}{C_M}, \quad C_M = c_D{}^2 + C_3{}^2. \tag{8}$$

Boundary condition (2e) has been used to obtain the right-hand sides of equations (7a, b) in terms of $\bar{v}(x, 0, p)$ and $\bar{v}_{,x}(x, 0, p)$.

The general solutions of equation (7) are given by

$$\begin{aligned}\bar{\bar{u}}(x, s, p) = {} & B_1(s, p)e^{\gamma_1 x} + B_2(s, p)e^{-\gamma_1 x} \\ & + B_3(s, p)e^{\gamma_2 x} + B_4(s, p)e^{-\gamma_2 x} \\ & + B_5(s, p)e^{\gamma_3 x} + B_6(s, p)e^{-\gamma_3 x} \\ & + T_3\int_{-\infty}^x \bar{v}(\eta, 0, p)\cos h[\gamma_3(\eta - x)]d\eta \\ & + T_2\int_{-\infty}^x \bar{v}(\eta, 0, p)\cos h[\gamma_2(\eta - x)]d\eta \\ & + T_1\int_{-\infty}^x \bar{v}(\eta, 0, p)\cos h[\gamma_1(\eta - x)]d\eta \\ & + t_3\int_{-\infty}^x \bar{\phi}(\eta, 0, p)\sin h[\gamma_3(\eta - x)]d\eta \\ & + t_2\int_{-\infty}^x \bar{\phi}(\eta, 0, p)\sin h[\gamma_2(\eta - x)]d\eta \end{aligned} \tag{9a}$$

$$\begin{aligned}\bar{\bar{v}}(x, s, p) = {} & -\frac{s}{\gamma_1}B_1(s, p)e^{\gamma_1 x} + \frac{s}{\gamma_1}B_2(s, p)e^{-\gamma_1 x} \\ & - \frac{\gamma_2}{s}B_3(s, p)e^{\gamma_2 x} + \frac{\gamma^2}{s}B_4(s, p)e^{-\gamma_2 x} \\ & - \frac{\gamma_3}{s}B_5(s, p)e^{\gamma_3 x} + \frac{\gamma_3}{s}B_6(s, p)e^{-\gamma_3 x} \\ & + V_3\int_{-\infty}^x \bar{v}(\eta, 0, p)\sin h[\gamma_3(\eta - x)]d\eta \\ & + V_2\int_{-\infty}^x \bar{v}(\eta, 0, p)\sin h[\gamma_2(\eta - x)]d\eta \\ & + V_1\int_{-\infty}^x \bar{v}(\eta, 0, p)\sin h[\gamma_1(\eta - x)]d\eta \\ & + V_3'\int_{-\infty}^x \bar{\phi}(\eta, 0, p)\cos h[\gamma_3(\eta - x)]d\eta \\ & + V_2'\int_{-\infty}^x \bar{\phi}(\eta, 0, p)\cos h[\gamma_2(\eta - x)]d\eta, \end{aligned} \tag{9b}$$

and

$$\begin{aligned}\bar{\bar{\phi}}(x, s, p) = {} & F_2B_3(s, p)e^{\gamma_2 x} \\ & + F_2B_4(s, p)e^{-\gamma_2 x} \\ & + F_3B_5(s, p)e^{\gamma_3 x} + F_3B_6(s, p)(s, p)e^{-\gamma_3 x} \\ & + W_3\int_{-\infty}^x \bar{v}(\eta, 0, p)\cos h[\gamma_3(\eta - x)]d\eta \\ & + W_2\int_{-\infty}^x \bar{v}(\eta, 0, p)\cos h[\gamma^2(\eta - x)]d\eta \\ & + W_1\int_{-\infty}^x \bar{v}(\eta, 0, p)\cos h[\gamma_1(\eta - x)]d\eta \\ & + W_3'\int_{-\infty}^x \bar{\phi}(\eta, 0, p)\cos h[\gamma_3(\eta - x)]d\eta \\ & + W_2'\int_{-\infty}^x \bar{\phi}(\eta, 0, p)\sin h[\gamma_2(\eta - x)]d\eta \end{aligned} \tag{9c}$$

where

$$\begin{aligned}
&\gamma_1{}^2 = s^2 + p^2, \\
&\gamma_2{}^2 = s^2 + \frac{p^2}{2\theta^2A^2}(A^2/C_2{}^2 + \theta^2B^2 - \theta^2A^2) + \frac{\epsilon}{j}\,\frac{(2A^2 - C^2)}{2\theta^2A^2} - R^{1/2}, \\
&\gamma_3{}^2 = s^2 + \frac{p^2}{2\theta^2A^2}(A^2/C_2{}^2 + \theta^2B^2 - \theta^2A^2) + \frac{\epsilon}{j}\,\frac{(2A^2 - C^2)}{2\theta^2A^2} + R^{1/2}, \\
&R = [p^2(A^2/C_2{}^2 - \theta^2) + \epsilon/j(2A^2 - C^2)]^2 - 4\epsilon\theta^2C^2p^2/j, \\
&T_1 = -\frac{1}{p^2}(-\gamma_1{}^2L^2 + s^2), \\
&T_2 = -\frac{1}{\theta^2A_2}\,\frac{1}{\gamma_3{}^2 - \gamma_2{}^2}\left(R_5\gamma_2 + R_5\gamma_1{}^2 + R_3 + \frac{T}{\gamma_2{}^2 - \gamma_1{}^2}\right), \\
&T_3 = L^2 - T_2 - T_1, \\
&t_2 = \frac{\theta^2S^2C^2}{\gamma_3}\,\frac{1}{\gamma_3{}^2 - \gamma_2{}^2}\,\frac{1}{\theta^2A^2}, \\
&t_3 = \frac{\theta^2S^2C^2}{\gamma_3}\,\frac{1}{\gamma_3{}^2 - \gamma_2{}^2}\,\frac{1}{\theta^2A^2}, \\
&V_1 = \frac{1}{s}\left[\gamma_1T_1 - \gamma_1L^2 + \frac{s^2}{\gamma_1}\right], \\
&V_i = \gamma_iT_i/s \quad (i = 2, 3), \\
&V'_i = \gamma_iT_i/s \quad (i = 2, 3), \\
&W_i = -\{\gamma_i{}^2T_i - (A^2s^2 + p^2)T_i - \gamma_iB^2sV_i\}1sC^2 \\
&\qquad (i = 1, 2, 3), \\
&W'_i = -\{\gamma_i{}^2t_i - (A^2s^2 + p^2)t_i - \gamma_iB^2sV_i^1\}1sC^2 \\
&\qquad (i = 2, 3), \\
&R_5 = \theta^2A^2L^2, \\
&L^2 = B^2 + C^2 - A^2, \\
&R_3 = \theta^2s^2(B^2 - L^2 - A^2L^2) - p^2L^2\left[\theta^2 + \frac{A^2}{C_2{}^2}\right] - \frac{\epsilon L^2}{j}(2A^2 - C^2), \\
&R_1 = \theta^2s^4(L^2 - B^2) + s^2p^2\left[\theta^2L^2 + \frac{L^2 - B^2}{C_2{}^2}\right] + \frac{L^2}{C_2{}^2}p^4 + \frac{\epsilon s^2}{j}(2L^2 - C^2) + \frac{2\epsilon}{j}L^2p^2, \\
&F_i = (-B^2s^2 - A^2\gamma_i{}^2 + \gamma_1{}^2)/sC^2 \quad (i = 2, 3).
\end{aligned} \tag{10}$$

The coefficients $B_i (i = 1, 2, \cdots, 6)$ are determined from the following conditions:

$$\left.\begin{aligned}\bar{\bar{u}}(x, s, p) = 0 \\ \bar{\bar{v}}(x, s, p) = 0 \\ \bar{\bar{\phi}}(x, s, p) = 0\end{aligned}\right\} \text{ at } x = \pm\infty. \tag{11}$$

We get

$$B_1(s, p) = -\frac{T_1}{2}\int_{-\infty}^{\infty}\bar{v}(\eta, 0, p)\, e^{-\gamma_1\eta} d\eta,$$
$$B_3(s, p) = -\frac{T_2}{2}\int_{-\infty}^{\infty}\bar{v}(\eta, 0, p)\, e^{-\gamma_2\eta} d\eta, + \frac{t_2}{2}\int_{-\infty}^{\infty}\bar{\phi}(\eta, 0, p)\, e^{-\gamma_1\eta} d\eta,$$
$$B_5(s, p) = -\frac{T_3}{2}\int_{-\infty}^{\infty}\bar{v}(\eta, 0, p)\, e^{-\gamma_3\eta} d\eta, + \frac{t_3}{2}\int_{-\infty}^{\infty}\bar{\phi}(\eta, 0, p)\, e^{-\gamma_3\eta} d\eta,$$
$$B_2(s, p) = B_4(s, p) = B_6(s, p) = 0. \tag{12}$$

The stress components in the transform space, $\bar{\bar{t}}_{xx}$, $\bar{\bar{t}}_{yy}$, $\bar{\bar{t}}_{yx}$, $\bar{\bar{M}}_{xz}$ and $\bar{\bar{M}}_{yz}$ can then be obtained by transforming the stress-displacement equations and substituting for $\bar{\bar{u}}$, $\bar{\bar{v}}$ and $\bar{\bar{\phi}}$ from equations (9). However, only $\bar{\bar{t}}_{yy}$ and $\bar{\bar{M}}_{yz}$ are considered since the main object of this study is the determination of the dynamic stress and couple stress intensity factors. The transformed, normalized stress component $\bar{\bar{t}}_{yy}$ is given by

$$\bar{\bar{t}}_{yy}(x, s, p) = \frac{1}{C_2^2}[-\bar{v}(x, 0, p) + s\bar{\bar{v}}(x, s, p)] + \frac{L^2}{C_2^2}\bar{\bar{u}}_{,x}. \tag{13}$$

Substituting $\bar{\bar{u}}$ and $\bar{\bar{v}}$ from equations (9a, b) into the equation (13) and integrating by parts, we obtain

$$\begin{aligned}\bar{\bar{t}}_{yy}(x, s, p) = &-\frac{T_1}{2}(L^2 - s^2/\gamma_1^2)\int_x^{\infty}\bar{v}_{,\eta}(\eta, 0, p)\, e^{-\gamma_1(\eta - x)} d\eta \\ &+\frac{T_1}{2}(L^2 - s^2/\gamma_1^2)\int_{-\infty}^{x}\bar{v}_{,\eta}(\eta, 0, p)\, e^{\gamma_1(\eta - x)} d\eta \\ &-\frac{T_2}{2}(L^2 - 1)\int_x^{\infty}\bar{v}_{,\eta}(\eta, 0, p)\, e^{-\gamma_2(\eta - x)} d\eta \\ &+\frac{T_2}{2}(L^2 - 1)\int_{-\infty}^{x}\bar{v}_{,\eta}(\eta, 0, p)\, e^{\gamma_2(\eta - x)} d\eta \\ &-\frac{T_3}{2}(L^2 - 1)\int_x^{\infty}\bar{v}_{,\eta}(\eta, 0, p)\, e^{-\gamma_3(\eta - x)} d\eta \\ &+\frac{T_3}{2}(L^2 - 1)\int_{\infty}^{x}\bar{v}_{,\eta}(\eta, 0, p)\, e^{\gamma_3(\eta - x)} d\eta \\ &+\frac{t_2\gamma_2}{2}(L^2 - 1)\int_x^{\infty}\bar{\phi}(\eta, 0, p)\, e^{-\gamma_2(\eta - x)} d\eta \\ &-\frac{t_2\gamma_2}{2}(L^2 - 1)\int_{-\infty}^{x}\bar{\phi}(\eta, 0, p)\, e^{\gamma_2(\eta - x)} d\eta \\ &+\frac{t_3\gamma_3}{2}(L^2 - 1)\int_x^{\infty}\bar{\phi}(\eta, 0, p)\, e^{-\gamma_3(\eta - x)} d\eta \\ &-\frac{t_3\gamma_3}{2}(L^2 - 1)\int_{-\infty}^{x}\bar{\phi}_{,\eta}(\eta, 0, p)\, e^{-\gamma_3(\eta - x)} d\eta \\ &-\frac{p^2}{\gamma_1^2}\bar{v}(x, 0, p).\end{aligned} \tag{14}$$

The inverse Fourier cosine transform for any point in the space is then taken and y is allowed to approach zero. Since $\bar{t}_{yy}(x, y, p)$ is continuous in y at $y = 0$, we can interchange the order of the limit process and integration in s. The inverse Fourier cosine transform which is defined by

$$\bar{f}(x, y, p) = \frac{2}{\pi}\int_0^{\infty}\bar{\bar{f}}(x, s, p)\cos(sy)\, ds \tag{15}$$

is performed on equation (14) for $y = 0$. The inverse Laplace transform is then taken to obtain the expression for t_{yy}. In view of the lengthy expressions for $\gamma_i(s, p)$, $i = 1, 2, 3$, we find it reasonable to use an approximation scheme for the inversion of the Laplace transform that would be unaffected by the singularities present in the integal formulation of t_{yy}, as will be seen later. We find it necessary to consider the case of large p, which corresponds to small t, based on the Tauberian theorems. The inverse Laplace transform is then taken by application of the Cagniard-Dehoop method and a convolution integral, that is, the expression for $t_{yy}(x, 0, p)$ obtained above is changed into a recognizable Laplace transform by setting $\gamma_i|\eta - x| = \mathrm{pt}$ ($i = 1, 2, 3$) in each double integral. Then, using the identity

$$\overline{\left[\frac{\partial}{\partial t}f * g(t)\right]} = p\bar{f}\bar{g}, \tag{16}$$

where $*$ denotes the convolution integral, we obtain

$$\begin{aligned}t_{yy}(x, 0, t) = &\frac{\partial}{\partial t}\Big[\int_{-\infty}^{\infty}\int_0^t v_{,\eta}(\eta, 0, \tau) H(t - \tau - |\eta - x|) \\ &M_1(t - \tau, \eta - x)\, d\tau d\eta\Big] \\ &+\int_{-\infty}^{\infty}\int_0^t v_{,\eta}(\eta, 0, \tau) H(t - \tau - |\eta - x|/A) \\ &M_2(t - \tau, \eta - x)\, d\tau d\eta \\ &+\int_{-\infty}^{\infty}\int_0^t v_{,\eta}(\eta, 0, \tau) H(t - \tau - |\eta - x|/\theta C_2) \\ &M_3(t - \tau, \eta - x)\, d\tau d\eta \\ &+\int_{-\infty}^{\infty}\int_0^t \phi(\eta, 0, \tau) H(t - \tau - |\eta - x|/A) \\ &M_4(t - \tau, \eta - x)\, d\tau d\eta \\ &+\int_{-\infty}^{\infty}\int_0^t \phi(\eta, 0, \tau) H(t - \tau - |\eta - x|/\theta C_2) \\ &M_5(t - \tau, \eta - x)\, d\tau d\eta\end{aligned} \tag{17}$$

where

$$\begin{aligned}M_1(\xi, \zeta) = &\frac{2}{\pi C_2^2}\Big[(1 - L^2)\frac{\xi}{\zeta} - \frac{1}{2}\frac{\zeta}{\xi} - \frac{(L^2 - 1)^2}{2}\frac{\xi^3}{\zeta^3}\Big][\xi^2 - \zeta^2]^{-1/2}, \\ M_2(\xi, \zeta) = &\frac{(L^2 - 1)^2}{\pi C_2^2}\Big[\frac{\xi}{\zeta^3}\Big(\xi^2 - \frac{\zeta 2^{1/2}}{A^2}\Big) \\ &+\frac{A^2\epsilon}{j}\frac{2 + C^2/B^2}{(A^2/C_2^2 - \theta_2)}\frac{1}{\zeta^3} \\ &\Big\{\frac{\xi(\xi^2 - \zeta^2/A^2)^{3/2}}{12} - \frac{\zeta^2\xi(\xi^2 - \zeta^2/A^2)^{1/2}}{8A^2} \\ &+\frac{\zeta^4}{8A_4}\ln\frac{\xi + (\xi^2 - \zeta^2/A^2)^{1/2}}{\zeta/A}\Big\}\Big], \\ M_3(\xi, \zeta) = &-\frac{(L^2 - 1)^2}{\pi C_2^2}\frac{A^2\epsilon}{j}\frac{2 + C^2/B^2}{(A^2/C_2^2 - \theta^2)}\frac{1}{\zeta^3} \\ &\times\Big[\frac{\xi(\xi^2 - \zeta^2/\theta_2 C_2^2)^{3/2}}{12} \\ &-\frac{\zeta^2\xi(\xi^2 - \zeta^2/\theta^2 C_2^2)^{1/2}}{8\theta^2 C_2^2} \\ &+\frac{\zeta^4}{8(\theta C_2)^4}\ln\frac{\xi + (\xi^2 - \zeta^2/\theta^2 C_2^2)^{1/2}}{\zeta/\theta C_2}\Big], \\ M_4(\xi, \zeta) = &\frac{(L^2 - 1)}{\pi}\frac{\theta^2 C^2}{A^2 - \theta^2 C_2^2}\frac{\xi}{\zeta^3}(\xi^2 - \zeta^2/A^2)^{1/2}, \\ M_5(\xi, \zeta) = &-\frac{(L^2 - 1)}{\pi}\frac{\theta^2 C^2}{A^2 - \theta^2 C_2^2}\frac{\xi}{\zeta^3} \\ &(\xi^2 - \zeta^2/\theta^2 C_2^2)^{1/2}\end{aligned} \tag{18}$$

and $H(t)$ is the Heaviside function. Equation (17) is rearranged in such a way that the terms with the Cauchy kernel $(\eta - x)^{-1}$ are extracted out and the terms with the kernel $(\eta - x)^{-3}$ in M_i ($i = 1, 2, \cdots, 5$) are combined so that the strong singularities across $\eta = x$ are cancelled. Noting that the $v_{,\eta}(\eta, 0, \tau)$ is equal to the macrorotation of the crack surface,

$\omega(\eta, 0, \tau)$, that following equation is obtained:

$$t_{yy}(x, 0, t)=\frac{2(1-L^2)}{\pi C_2^2}\frac{\partial}{\partial t}J(x, t), \tag{19}$$

where

$$\begin{aligned} J(x, t)= & \\ & -\frac{(1-L^2)}{2}\iint_{A1-A2}\omega(\eta, 0, \tau)\frac{(t-\tau)^2}{(\eta-x)^3}d\tau d\eta \\ & +[1-(1-L^2)/4]\iint_{A1}\omega(\eta, 0, \tau)\frac{1}{(\eta-x)}d\tau d\eta \\ & -\frac{(1-L^2)}{4A^2}\iint_{A2}\omega(\eta,0, \tau)\frac{1}{(\eta-x)}d\tau d\eta \\ & +\frac{1}{12}\frac{1-L^2}{2}\frac{A^2\epsilon}{j}\frac{2+C^2/B^2}{(A^2/C_2^2-\theta^2)}\iint_{A2-A3} \\ & \left\{\omega(\eta, 0, \tau)\frac{(t-\tau)^4}{(\eta-x)^3}\right\}d\tau d\eta \\ & -\frac{1}{4}\frac{1-L^2}{2}\frac{A^2\epsilon}{j}\frac{2+C^2B^2}{(A^2/C_2^2-\theta^2)}\iint_{A2} \\ & \left\{\frac{\omega(\eta, 0, \tau)}{A^2}\frac{(t-\tau)^2}{(\eta-x)}\right\}d\tau d\eta \\ & +\frac{1}{4}\frac{1-L^2}{2}\frac{A^2\epsilon}{j}\frac{2+C^2/B^2}{(A^2/C_2^2-\theta_2)}\iint_{A3} \\ & \left\{\frac{\omega(\eta, 0, \tau)}{\theta_2 C_2^2}\frac{(t-\tau)^2}{(\eta-x)}\right\}d\tau d\eta \\ & +\iint_{A1}\omega(\eta, 0, \tau)K_1(t-\tau, \eta-x) \\ & \times[(t-\tau)^2-(\eta-x)^2]^{1/2}d\tau d\eta \\ & +\iint_{A2}\omega(\eta, 0, \tau)K_2(t-\tau, \eta-x)d\tau d\eta \\ & +\iint_{A3}\omega(\eta, 0, \tau)K_3(t-\tau, \eta-x) \\ & -\frac{\pi}{2(1-L^2)}\int_{-1-a_-(t)}^{x}\omega(\eta, 0, t)d\eta \\ & -\frac{\theta^2 C_2^2 C_2^2}{2(A^2-\theta^2 C_2^2)}\iint_{A2-A3}\phi(\eta, 0, \tau)\frac{(t-\tau)^2}{(\eta-x)^3}d\tau d\eta \\ & +\frac{\theta_2 C_2^2 C^2}{4A^2(A^2-\theta^2 C^2)}\iint_{A2}\phi(\eta, 0, \tau)\frac{1}{(\eta-x)}d\tau d\eta \\ & +\frac{\theta^2 C_2^2 C^2}{4\theta^2 C_2^2(A^2-\theta^2 C^2)}\iint_{A3}\phi(\eta, 0, \tau)\frac{1}{(\eta-x)}d\tau d\eta \\ & +\iint_{A2}\phi(\eta, 0, \tau)K_4(t-\tau, \eta-x)d\tau d\eta \\ & +\iint_{A3}\phi(\eta, 0, \tau)K_5(t-\tau, \eta-x)d\tau d\eta, \end{aligned} \tag{20}$$

where

$$\begin{aligned} K_1(\xi, \zeta)= & \frac{\zeta}{\xi}\Big[\frac{1}{1+(1-\zeta^2/\xi^2)^{1/2}}-\frac{1}{2(1-L^2)} \\ & -\frac{(1-L^2)}{8}\frac{3+\zeta^2/\xi^2}{1+(1+0.5\zeta^2/\xi^2)(1-\zeta^2/\xi^2)^{1/2}}\Big], \\ K_2(\xi, \zeta)= & -\frac{(1-L^2)}{4A^2}\frac{1}{\xi^2}\{1+q_1^{1/2}\}^{-2} \\ & +\frac{1}{12}\frac{1-L^2}{2}\frac{A^2\epsilon}{j}\frac{2+C^2/B^2}{(A^2/C_2^2-\theta^2)}\frac{\xi}{A^4} \\ & \Big[\{1.5+q_1^{1/2}\}^{-1}-0.25\{1.5+q_1^{1/2}\}^{-1}\{1+q_1^{1/2}\}^{-2}\Big] \\ & +\frac{1}{8}\frac{1-L^2}{2}\frac{A^2\epsilon}{j}\frac{2+C_2/B^2}{(A^2/C_2^2-\theta^2)}\frac{\zeta}{A^4} \\ & \Big[\{1+q_1^{1/2}\}^{-1}+\ln\frac{\xi+(\xi^2-\zeta^2/A^2)^{1/2}}{\zeta/A}\Big], \\ K^3(\xi, \zeta)= & -\frac{1}{12}\frac{1-L^2}{2}\frac{A^2\epsilon}{j}\frac{2+C^2/B^2}{(A^2/C_2^2-\theta^2)}\frac{\xi}{\theta^4 C_2^4} \\ & \times\Big[\{1.5+q_2^{1/2}\}^{-1}-0.25\{1.5+q_2^{1/2}\}^{-1}\{1 \\ & +q_2^{1/2}\}^{-2}\Big] \\ & -\frac{1}{8}\frac{1-L^2}{2}\frac{A^2\epsilon}{j}\frac{2+C^2/B^2}{(A^2/C_2^2-\theta^2)}\frac{\zeta}{\theta^4 C^4{}_2} \\ & \Big[\{1+q_2^{1/2}\}^{-1}+\ln\frac{\xi+(\xi^2-\zeta^2/A_2)^{1/2}}{\zeta/\theta C_2}\Big], \\ K_4(\xi, \zeta)= & \frac{\theta^2 C_2^2 C^2}{4A^4(A^2-\theta^2 C_2^2)}\frac{\zeta}{\xi^2}\{1+q_1^{1/2}\}^{-2}, \\ K^5(\xi, \zeta)= & \frac{\theta^2 C^2 C_2^2}{4\theta^4 C_2^4(A^2-\theta^2 C_2^2)}\frac{\zeta}{\xi^2}\{1+q_2^{1/2}\}^{-2} \\ q_1= & \Big[1-\frac{1}{A^2}\frac{\zeta^2}{\xi^2}\Big], \\ q_2= & \Big[1-\frac{1}{\theta^2 C_2^2}\frac{\zeta^2}{\xi^2}\Big]; \\ A_1= & \{(\eta, \tau)|0<\tau<t-|\eta-x|, \\ & -1-a_-(\tau)<\eta<1+a_+(\tau)\}, \\ A_2= & \{(\eta, \tau)|0<\tau<t-|\eta-x|/A, \\ & -1-a_-(\tau)<\eta<1+a_+(\tau)\}, \\ A_3= & \{(\eta, \tau)|0<\tau<t-|\eta-x|/\theta C_2, \\ & -1-a_-(\tau)<\eta<1+a_+(\tau)\} \end{aligned} \tag{21}$$

Likewise, the transformed, normalized couple stress, $\bar{\bar{M}}_{yz}(x, s, p)$ is given by

$$\bar{\bar{M}}_{yz}(x, s, p)=j\theta^2[-\bar{\phi}(x, 0, p)+s\bar{\bar{\phi}}]. \tag{22}$$

Substituting for $\bar{\bar{\phi}}$ from equation (9c) into equation (22), and integrating by parts, we obtain

$$\begin{aligned} \bar{\bar{M}}_{yz}(x, s, p)= & -\frac{sW_3}{2}\int_x^\infty \bar{v}(\eta, 0, p)e^{-\gamma_3(\eta-x)}d\eta \\ & +\frac{sW_3}{2}\int_x^\infty \bar{v}(\eta, 0, p)e^{\gamma_3(\eta-x)}d\eta \\ & +\frac{s^2}{2\gamma_3^2}\int_x^\infty \bar{\phi}, \eta(\eta, 0, p)e^{-\gamma_3(\eta-x)}d\eta \\ & -\frac{s^2}{2\gamma_3^2}\int_{-\infty}^x \bar{\phi}, \eta(\eta, 0, p)e^{\gamma_3(\eta-x)}d\eta. \end{aligned} \tag{23}$$

By following a method similar to that used for $t_{yy}(x, 0, t)$, we obtain

$$M_{yz}(x, 0, t)=\frac{j\theta^2}{\pi}\frac{\partial}{\partial t}J'(x, t) \tag{24}$$

where

$$\begin{aligned} J'(x, t)= & \iint_{A3}\phi,_\eta(\eta, 0, \tau)\frac{1}{(\eta-x)}d\tau d\eta \\ & +\iint_{A3}\phi,_\eta(\eta, 0, \tau)K_6(t-\tau, \eta-x)d\tau d\eta \\ & -(1-L^2)\frac{A^2\epsilon}{j}\frac{1}{C^2\theta^2}(2+C^2/B^2) \\ & \times\Big[\iint_{A3}v(\eta, 0, \tau)\frac{(t-\tau)^2}{(\eta-x)^3} \\ & -\frac{1}{2\theta^2 C_2^2}\iint_{A3}v(\eta, 0, \tau)\frac{1}{(\eta-x)}d\tau d\eta \\ & +\iint_{A3}v(\eta, 0, \tau)K_7(t-\tau, \eta-x)d\tau d\eta\Big], \end{aligned} \tag{25}$$

and where

$$\begin{aligned} K^6(\xi, \zeta)= & -\frac{\zeta}{\theta^2 C_2^2\xi^2}[1+q_2^{1/2}]^{-1} \\ K^2(\xi, \zeta)= & -\frac{\zeta}{2\theta 4_2^4\xi^2}[1+q_2^{1/2}]^{-2}. \end{aligned} \tag{26}$$

The η-integration in equations (20) and (25), which include the Cauchy kernels, is performed in the sense of the Cauchy principal value, if they do not exist in the sense of Riemann. Recalling that $t_{yy}(x, 0, t)$ is given as a boundary condition on the crack surface, equation (19) can be viewed as an integro-differential equation for the unknown functions $\omega(\eta, 0, \tau)$ and $\phi(\eta, 0, \tau)$. Like wise, $M_{yz}(x, 0, t)$ is given as a boundary condition on the crack surface and equation (24) can be viewed as another integro-differenctial equation for the unknown functions $v(\eta, 0, \tau)$ and $\phi, \eta(\eta, 0, \tau)$. Moreover, $v(\eta, 0, \tau)$ can be obtained from $\omega(\eta, 0, \tau)$ and $\phi(\eta, 0, \tau)$ from $\phi,_\eta(\eta, 0, \tau)$. However, the evaluation of $\omega(\eta, 0, \tau)$ and $\phi,_\eta(\eta, 0, \tau)$ can only be carried out numerically since the equations are not analytically tractable. In order to facilitate the application of numerical techniques, equations (19) and (24) are changed into integral equation from by integration with respect to time. Note that

$$\int_0^{t_c} t_{yy}(x, 0, \tau)\, d\tau = \frac{2(1-L^2)}{\pi C_2^2} J(x, t_c), \text{ for } |x|>1, \tag{27}$$

where t_c is the time for the propagating crack tip to arrive at x, such that $x=1+a_+(t_c)$ for $x>1$ and $x=-1-a_-(t_c)$ for $x<-1$. Then, we obtain

$$\frac{2(1-L^2)}{\pi C_2^2}[J(x, t)-H(|x|-1)J(x, t_c)] = \int_{t_c}^{t} t_{yy}(x, 0, \tau)\, d\tau \tag{28}$$

and

$$[J'(x, t)-H(|x|-1)J'(x, t_c)]=0 \tag{29}$$

In equations (28) and (29), it is assumed that

$$t_c=0, \text{ if } |x|<1.$$

In order to evaluate $\omega(\eta, 0, \tau)$ and $\phi,_\eta(\eta, 0, \tau)$, present in equations (28) and (29), it is expedient to cast them in a form in which their singularities are explicit. In view of previous analyses in the literature, two types of singularities can be expected. The first type is the square root signularity which arises at the crack tip, the proof of which can be found in Achenbach and Bazant(1975) and Freund and Clifton(1974). The second type is the traveling logarithmic sigularity located at the front of the Rayleigh wave, confirmed by Baker(1962) and Thau and Lu(1971), each of whom differentiated the normal displacement of the crack surface with respect to the coordinates of the crack propagation direction. Despite the fact that $\omega=v,_x$ has a lograrithmic singularity as $x\rightarrow \pm C_R t$(see the appendix in Kim, (1977), it turns out that the final evaluation of the normal stress component t_{yy} has only a square root singularity according to the analysis of (Kim, 1977). In the present case, of micropolar medium, the analysis for $\phi,_\eta$ is quite analogous to that for the macrorotation ω. We now express ω and $\phi,_\eta$ as

$$\omega(\eta, 0, \tau) = \frac{\Omega(\eta, \tau)}{[1+a_+(\tau)-\eta]^{1/2}[1+a_2(\tau)+\eta]^{1/2}} \tag{30a}$$

and

$$\phi,_\eta(\eta, 0, \tau) = \frac{\Phi(\eta, \tau)}{[1+a_+(\tau)-\eta]^{1/2}[1+a_-(\tau)+\eta]^{1/2}}, \tag{30b}$$

where Ω and Φ are assumed to be bounded and continuous almost everywhere over the area, S, defined by $S=\{(\eta, \tau)|\tau \geq 0, -1-a_-(\tau)|<\eta<1+a_+(\tau)\}$. The functions Ω and Φ are zero if (η, τ) is not in S. Therefore, there is a jump discontinuity in Ω and Φ across the crack tip trajectories. In order to derive formulas for the stress and couple stress intensity factors, it is further assumed that Ω and Φ are analytic in the region S almost everywhere along the crack tip trajectories. Moreover, in the numerical integration process, Ω and Φ are treated as if they are bounded and continuous everywhere in S.

4. STRESS AND COUPLE STRESS INTENSITY FACTORS

The dynamic stress intensity factor $K_{ID}^{\pm}(t)$ is defined by

$$K_{ID}^{\pm}(t) = \lim_{\delta\rightarrow 0^+}\sqrt{2\pi\delta}\, t_{yy}(\pm 1 \pm a_{\pm}(t) \pm \delta, t), \tag{31}$$

where the upper and lower signs are, respectively, for the right and left crack tips. The evaluation of $K_{ID}^{\pm}(t)$ is then considered for the case of uniform extension of a crack. The formula obtained for this case will be extended to nouniform propagation of the crack without a detailed proof. For uniform extension, equation (30a) is written as

$$\omega(\eta, 0, \tau) = \frac{\Omega(\eta, \tau)}{[(1+c\tau)^2-\eta^2]^{1/2}} \tag{32}$$

The procedure for obtaining $K_{ID}^{\pm}(t)$ has been described, where c has already been defined as the crack tip velocity. This procedure is analogous to the one used by Kim(1977) for the classical case, which we will not repeat here. However, we point out that the procedure consists of choosing a particular point (x, t) in the η-τ plane such that $x>1+ct$ and constructing the regions A_1, A_2, and A_3 by drawing the characteristic lines. Then by introducing an appropriate polar coordinate system and following the analysis of Muskhelishvili[(1953), equation (29.3)], we evaluate $K_{ID}^{+}(t)$:

$$K_{ID}^{+}(t) = f(A, C_2, c)\ \Omega(1+ct, t)/\sqrt{1+ct}, \tag{33}$$

where

$$\begin{aligned} f(A, C_2, C) = &-\frac{2(1-L^2)\pi^{1/2}}{C_2^2}[1-0.25(1-L^2)] \\ &+\frac{(L^2-1)^2\pi^{1/2}}{2A^2C_2^2} - \frac{2(1-L^2)c^2\pi^{1/2}}{(1-c^2)^{1/2}C_2^2}\Bigg[\frac{1}{1+(1-c^2)^{1/2}} \\ &-\frac{1}{2(1-L^2)} \\ &-\frac{(1-L^2)}{8}\frac{3+c^2}{[1+(1+0.5c^2)(1-c^2)^{1/2}]}\Bigg] \\ &+\frac{(L^2-1)2\pi^{1/2}}{2A^2c_2^2}\frac{c^2}{A^2}[1+\sqrt{1+c^2/A^2}]^{-2} \end{aligned} \tag{34}$$

for the right crack tip. In a similar manner, we obtain

$$K_{ID}^{-}(t) = -f(A, C_2, c)\ \Omega(-1-ct, t)/\sqrt{1+ct} \tag{35}$$

for the left crack tip.

Recalling that $K_{ID}^{\pm}(t)$ was determined by considering the neighborhood of the point, $(1+ct, t)$, in the $\eta-\tau$ plane, the applicability of equations (33) and (35) can be extended to the case of uniform rates of propagation with some modification.

The following equation is then obtained:

$$K_{ID}^{\pm}(t)=\pm f[A,\,C_2,\,c_{\pm}(t)]\times\frac{\Omega[\pm1\pm a_{\pm}(t),\,t]}{[\{2+a_{-}(t)+a_{+}(t)/2\}]^{1/2}} \tag{36}$$

Similarly, the dynamic couple stress intensity factor $K_{ID}^{\pm}(t)$ is defined by

$$K_{ID}^{\pm}=\lim_{\delta\to0^+}\sqrt{2\pi\delta}/b\;\,M_{yz}(\pm1\pm a_{\pm}(t)\pm\delta,\,t), \tag{37}$$

where the characteristic length b is given by

$$b^2=\frac{\gamma}{2(2\mu+\kappa)}$$

and the upper and lower signs are, respectively, for the right and left crack tips.

For uniform extension, equation (30b) is written as

$$\phi,_{\eta}(\eta,\,0,\,\tau)=\frac{\Phi(\eta,\,\tau)}{[(1+c\tau)^2-\eta^2]^{1/2}}. \tag{38}$$

Then, according to the definition,

$$K_{ID}^{+}(t)=g(\theta C_2,\,c)\;\,\Phi(1+ct,\,t)/\sqrt{1+ct} \tag{39}$$

for the right crack tip and

$$g(\theta C_2,\,c)=-j\theta^2/b\pi\times\left[1-\frac{c^2}{\theta_2C_2{}^2}[1+\{1+\{1-c^2/\theta^2C_2{}^2\}^{1/2}\}]^{-1}\right]. \tag{40}$$

In a similar manner, we obtain

$$K_{IM}^{-}(t)=-g(\theta C_2,\,c)\;\,\Phi(-1-ct,\,t)/\sqrt{1+ct} \tag{41}$$

for the left crack tip and

$$K_{IM}^{\pm}(t)=\pm g[\theta C_2,\,c_{\pm}(t)]\times\frac{\Phi[\pm1\pm a_{\pm}(t),\,t]}{[\{2+a_{-}(t)+a_{+}(t)\}/2]^{1/2}} \tag{42}$$

In orfer to obtain $K_{ID}^{\pm}(t)$ and $K_{IM}^{\pm}(t)$, $\Omega[\pm1\pm a_{\pm}(t)]$ and $\Phi[\pm1\pm a_{\pm}(t)]$ are first obtained by solving the simultaneous integral equations, (28) and (29), numerically. Then $K_{ID}^{\pm}(t)$ and $K_{ID}^{\pm}(t)$ are computed by using equations (36) and (42).

5. EXAMPLE PROBLEM

As a numerical example, the diffraction of a uniform micropolar dilatational wave with a propagation vector normal to the crack plane for the case of a stationary crack is investigated. The total wave field for a diffraction problem is determined by adding the incident wave field and the scattered wave field. For the purpose of determining the dynamic stress and couple stress intensity factors, only the scattered wave field must be considered. The boundary conditions (2a) and (2b) for the scattered wave field are given, respectively, by

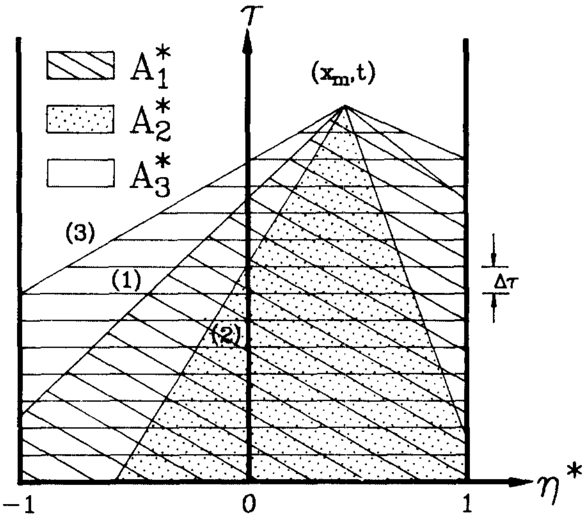

Fig. 2 Division of the area of integration for numerical integration:
(1) $|\eta^*-X_m|=t-\tau$,
(2) $|\eta^*-X_m|=A(t-\tau)$,
(3) $|\eta^*-X_m|=\theta C_2(t-\tau)$

$$\left.\begin{array}{l} t_{yy}(x,\,0,\,t)=-\sigma H(t)\\ M_{yz}(x,\,0,\,t)=0\end{array}\right\}\quad\text{for } |x|<1, \tag{43}$$

where σ is the uniform pressure on the crack surface, and $a(t)=a_+(t)=a_-(t)$.

The numerical procedure for the computation of integral equation (29) is analogous to that of Kim (1977) and is outlined as follows: Introduce a new variable, defined by $\eta^*=\eta/[1+a(\tau)]$, and as a consequence the regions A_1, A_2, and A_3 are as a consequence the regions $A_1{}^*$, $A_2{}^*$, and $A_3{}^*$(see Fig. 2). Then, we divide $A_1{}^*$,$A_2{}^*$, and $A_3{}^*$ into a set of horizontal strips of equal spacing with interval $\Delta\tau$. As previously noted, the logarithmic singularities are neglected and Ω^* and Φ^* are approximated in each strip by

$$\Omega^*(\eta^*,\,\tau)=\sum_{j=1,3}^{2M-1}[a_{kj}+b_{kj}(\tau-\tau_k)]\,T_j(\eta^*), \tag{44a}$$

$$\Phi^*(\eta^*,\,\tau)=\sum_{j=1,3}^{2M-1}[c_{kj}+d_{kj}(\tau-\tau_k)]\,T_j(\eta^*), \tag{44b}$$

where

$$\Omega^*(\eta^*,\,\tau)=\Omega(\eta,\,\tau),\qquad \Phi^*(\eta^*,\,\tau)=\Phi(\eta,\,\tau),\qquad \tau_k\le\tau\le\tau_{k+1},$$

and a_{kj}, b_{kj}, c_{kj} and d_{kj}, are constants and T_j is the j^{th} order Chebyshev polynomial of the first kind. Note that only odd order polynomials are included in the terms related to macrorotation, ω, and microrotation, ϕ, due to the antisymmetry of both rotations with respect to the y-axis for the problem under consideration, and that only even order polynomials are included in the terms related to displacement, v, in the y-direction and the gradient of microrotation, $\phi,_{\eta}$. Both $\tau_1=0$ and $a_{1j}=c_{1j}=0$, are taken from the initial conditions. Then, from the continuity of Ω^* and Φ^* at $\tau=0$ and $a_{1j}=c_{1j}=0$, are taken from the initial conditions. Then, from the continuity of

Ω^* and Φ^* at $\tau=\tau_k$, we have $a_{kj}=a_{lj}+b_{lj}\Delta\tau$ and $c_{kj}=c_{lj}+d_{lj}\Delta\tau$, where $l=k-1$, and $\Delta\tau=\tau_k-\tau_{k-1}$. The problem is now reduced to the determination of b_{kj} and d_{kj} for each strip. In order to compute b_{ij} and $d_{ij}(\tau_1<\tau<\tau_{i+1}=t)$, pick M values of x, which are the zeros of $T_{2M}(x/[1+a(t)]$ in $[0, 1+a(t)]$, that is,

$$x_m=[1+a(t)]\cos\left(\frac{2m-1}{2M}\ \frac{\pi}{2}\right), \tag{45}$$

where $m=1, 2, \cdots, M$. Then, substituting equation (44a) into equation (28) and equation (44) into equation (29), the following $2M\times 2M$ simultaneous linear system of equations is obtained:

$$\sum_{k=1}^{i}\sum_{j=1,3}^{2M-1}[a_{kj}F_{j1k}(x_m, t)+b_{kj}F_{j2k}(x_m, t)]+\sum_{k=1}^{i}\sum_{j=1,3}^{2m-1}[c_{kj}G_{j1k}(x_m, t)+d_{kj}G_{j2k}(x_m, t)\Big]-\frac{\pi}{2(1-L^2)}\sum_{j=1,3}^{2M-1}[a_{kj}+b_{kj}(t-\tau_1)]\int_{-1}^{x_m}Qd\eta^*=-t \quad \text{for } |x_m|<1 \tag{46}$$

and

$$\sum_{k=1}^{i}\sum_{j=2,4}^{2M}[a_{kj}D_{j1k}(x_m, t)+b_{kj}D_{j2k}(x_m, t)]+\sum_{k=1}^{1}\sum_{j=2,4}^{2M}[c_{kj}H_{j1k}(x_m, t)+d_{kj}H_{j2k}(x_m, t)]=0 \quad \text{for } |x_m|<1, \tag{47}$$

where $m=1, 2, \cdots, M$,

$$\begin{aligned}F_{jlk}(x_m, t)=&-\frac{(1-L^2)}{2}\iint_{A_{1k}^*-A_{2k}^*}Q_f\frac{(t-\tau)^2}{(\eta^*-x)}d\tau d\eta^*\\&+[1-(1-L^2)/4]\iint_{A_{1k}^*}Q_f\frac{1}{(\eta^*-x)}d\tau d\eta^*\\&-\frac{(1-L^2)}{4A_2}\iint_{A_{2k}^*}Q_f\frac{1}{(\eta^*-x)}d\tau d\eta\\&+\frac{1-L^2}{24}\ \frac{A^2\epsilon}{j}\ \frac{2+C^2/B^2}{(A^2/C_2^2-\theta^2)}\iint_{A_{2k}^*-A_{3k}^*}Q_f\frac{(t-\tau)^4}{(\eta^*-x)^3}d\tau d\eta^*\\&-\frac{1-L^2}{8}\ \frac{A^2\epsilon}{j}\ \frac{2+C^2/B^2}{(A^2/C_2^2-\theta^2)}\iint_{A_{2k}^*}\frac{Q_f}{A_2}\ \frac{(t-\tau)^2}{(\eta^*-x)}d\tau d\eta^*\\&+\frac{1-L^2}{8}\ \frac{A^2\epsilon}{j}\ \frac{2+C^2/B^2}{(A_2/C_2^2-\theta^2)}\iint_{A_{3k}^*}\frac{Q_f}{\theta^2C_2^2}\ \frac{(t-\tau)^2}{(\eta^*-x)}d\tau d\eta^*\\&+\iint_{A_{2k}^*}Q_fK_2(t-\tau, \eta^*-x)d\tau d\eta^*\\&+\iint_{A_{3k}^*}Q_fK_3(t-\tau, \eta^*-x),\end{aligned} \tag{48}$$

$$\begin{aligned}G_{jlk}(x_m, t)=&-\frac{\theta^2C_2^2C^2}{2(A^2-\theta^2C_2^2)}\iint_{A_{2k}^*-A_{3k}^*}\phi^*f_{lk}(\tau)\frac{(t-\tau)^2}{(\eta^*-x)^3}d\tau d\eta^*\\&+\frac{\theta^2C_2^2C^2}{4A^2(A^2-\theta^2C_2^2)}\iint_{A_{2k}^*}\phi^*f_{lk}(\tau)\frac{1}{(\eta^*-x)}d\tau d\eta^*\\&+\frac{C^2}{4(A^2-\theta^2C_2^2)}\iint_{A_{3k}^*}\phi^*1_{lk}(\tau)\frac{1}{(\eta^*-x)}d\tau d\eta^*\\&+\iint_{A_{2k}^*}\phi^*f_{lk}(\tau)K_4(t-\tau, \eta^*-x)d\tau d\eta^*\\&+\iint_{A_{3k}^*}\phi^*f_{lk}(\tau)K_5(t-\tau, \eta^*-x)d\tau d\eta^*\end{aligned} \tag{49}$$

$$\begin{aligned}H_{jlk}(x_m, t)=&\iint_{A_{3k}^*}Q_f\frac{1}{(\eta^*-x)}d\tau d\eta^*\\&+\iint_{A_{3k}^*}Q_fK_6(t-\tau, \eta^*-x)d\tau d\eta^*\end{aligned} \tag{50}$$

and

$$\begin{aligned}D_{jlk}(x_m, t)=&-(1-L^2)\frac{A^2\epsilon}{jC_2\theta^2}(2+C^2/B^2)\\&\times\Big[\iint_{A_{3k}^*}v^*f_{lk}(\tau)\frac{(t-\tau)^2}{(\eta^*-x)^3}d\tau d\eta^*\\&-\frac{1}{2\theta C_2^2}\iint_{A_{3k}^*}v^*f_{lk}(\tau)\frac{1}{(\eta^*-x)}d\tau d\eta^*\\&+\iint_{A_{3k}^*}v^*f_{lk}(\tau)K_7(t-\tau, \eta^*-x)d\tau d\eta^*\Big],\end{aligned} \tag{51}$$

$$\begin{aligned}&Q=\frac{T_j(\eta^*)}{\sqrt{1-\eta^{*2}}},\\&Q=\frac{T_j(\eta^*)}{\sqrt{1-\eta^{*2}}}f_{lk}(\tau),\\&\phi^*=\int\frac{T_i(\eta^*)}{\sqrt{1-\eta^{*2}}}d\eta^* \qquad (j=2, 4, \cdots, 2M),\\&v^*=\int\frac{T_j(\eta^*)}{\sqrt{1-\eta^{*2}}} \qquad (j=1, 3, \cdots, 2M\text{-}1),\\&f_{1k}(\tau)=1, \qquad f_{2k}(\tau)=\tau-\tau_k.\end{aligned} \tag{52}$$

The areas A_{1k}^*, A_{2k}^*, and A_{3k}^* of the k^{th} strip are, respectively, associated with A_1^*, A_2^*, and A_3^*. The area intergtals F_{jlk}, G_{jlk} D_{jlk} and H_{jlk} are computed approximately by the application of quadrature formulas of the Gauss type, with the exception of the η^*-integrals in the second and third terms in F_{jlk} and G_{jlk}, the first and second terms in D_{jlk}, and the first term in H_{jlk}, which are computed analytically by the use of the recurrence formula, $T_{j+1}(x)=2xT_j(x)-T_{j-1}(x)$. The details of the integration procedure are described by Han(1989).

6. RESULTS AND DISCUSSION

The dynamic stress and the couple stress intensity factors given by equations (33) and (39) can be rewritten as follows:

$$K_{ID}^*(t)=\frac{\sqrt{\pi}C_2^2}{2(1-L^2)}f(A, C_2, c)\Omega^*(1, t), \tag{53}$$

$$K_{IM}^*(t)=\frac{\pi b}{j\theta^2}g(j\theta^2, c)\Phi^*(1, t). \tag{54}$$

where equation (53) is obtained by dividing equation (33) by $\sigma\sqrt{\pi}$ and equation (54) is obtained by multiplying equation (39) by $\pi b/j\theta^2$ and utilizing the boundary condition $t_{yy}(x, 0, \tau)=-\sigma=-2(1-L^2)/\pi C_2^2$[see equation (27)].

The micropolar coupling factor, N, is defined by

$$N^2=\frac{\kappa}{2(\mu+\kappa)}. \tag{55}$$

The coefficients of the unknown functions, $\Omega^*(\eta^*, t)$ and $\Phi^*(\eta^*, t)$, were computed by solving the simultaneous two-dimensional singular integral equations (46) and (47), from which the normalized dynamic stress and couple stress inten-

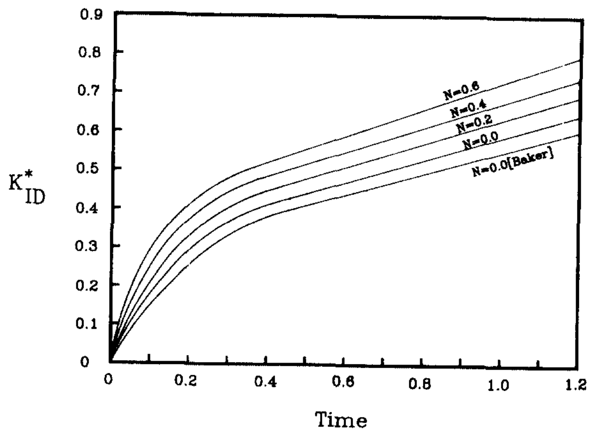

Fig. 3 Normalized dynamic stress intensity factor of a stationary crack for various coupling factors ($\nu=0.292$)

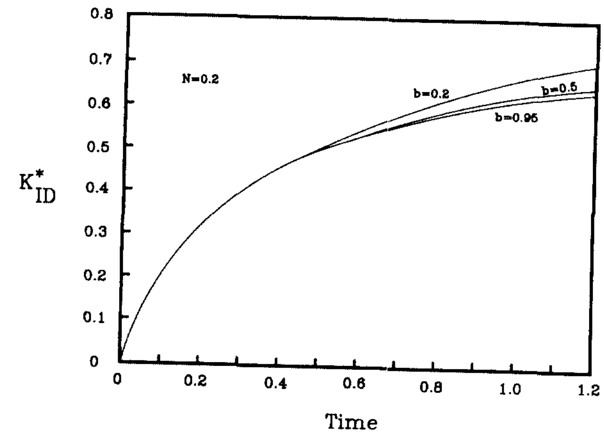

Fig. 5 Normalized dynamic stress intensity factor of a stationary crack for various characteristic lengths($\nu=0.292$)

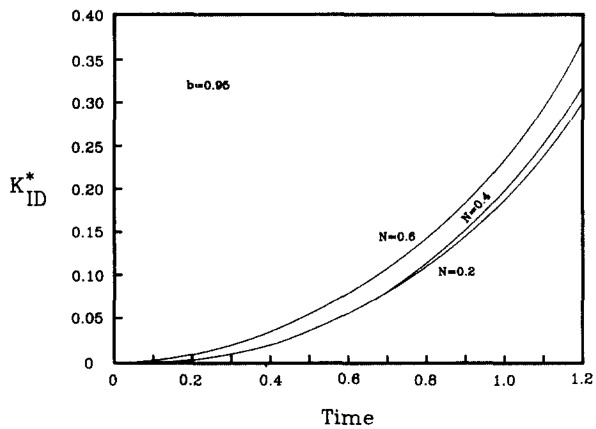

Fig. 4 Normalized dynamic couple stress intensity factor of a stationary crack for various coupling factors ($\nu=0.292$)

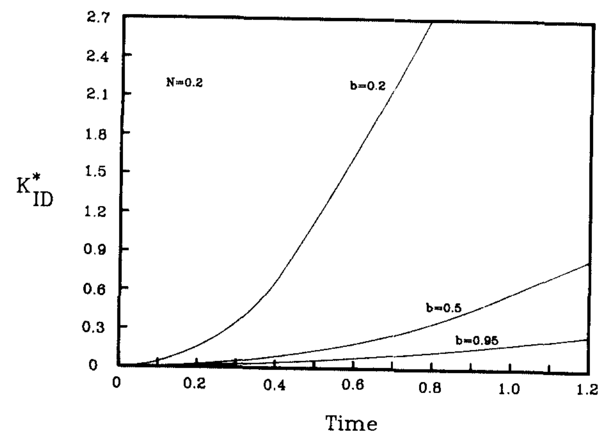

Fig. 6 Normalized dynamic couple stress intensity factor of a stationary crack for various characteristic lengths($\nu=0.292$)

sity factors were obtained by use of equations (53) and (54). The numerical results for $N=0$, 0.2, 0.4, 0.6, $b=0.95$ and $j=0.0196\text{mm}^2$ are shown in Figure 3, wihch is compared wth the work of Baker (1962), and Fig. 4 Our result is found to be within 5 percent of that of Baker. The numerical results for $b=0.2$, 0.5, 0.95, $N=0.2$ and $j=0.0196\text{mm}^2$ are shown is Fig. 5 and 6.

It is seen from Fig. 3 and 4 that both the micropolar dynamic stress and couple stress intensity factors increase as the coupling factor increases. From Fig. 5 and 6, the micropolar dynamic couple stress intensity factor is found to decrease as the characteristic length increases, although the micropolar dynamic stress intensity factor remains practically the same. Therefore, the micropolar stress intensity factor is always greater than the classical stress intensity factor. The characteristic length does not significantly effect the micropolar stress intensity factor, while it does affect the couple stress intensity factor, as along as the characteristic length for a finite crack is of the order of half of the crack length.

Equation (19) for stress involves ω and ϕ and equation (24) for couple stress involves both the displacement, transverse to the crack surface, and the gradient of microrotation. The classical solution for the corresponding problem has also been obtained as a special case of our micropolar solution by dropping the micropolar moduli in equation (46) and following the same numerical procedure. The corresponding curve is depicted in Figure 3 along with the exact solution of Baker(1982) for the purpose of comparison. Our result is found to be in good agreement with that of Baker.

However, the Laplace transform inversion procedure, based on the present problem, restricted the analysis to the normalized time range to about $t=1.2$. Although the solution for the times when the stress and couple stress intensity factors reached their maximum values was not obtained, useful information has been found to emerge from our solution concerning the behavior of the micropolar stress and couple stress distributions in the dynamic crack propagation process, the microrotation field and microinertia. Moreover, it is reasonable to expect that the behavior of the stress intensity factor for larger times would follow a similar pattern as that of the classical case for corresponding times, allowing for stress to reach maximum.

REFERENCES

Achenbach, J.D. and Bazant, Z.P., 1975, "Elastodynamic

Near-Tip Stress and Displacement Fields for Rapidly Propagating Cracks in Orthotropic Materials", Journal of Applied Mechanics, Vol. 42, p. 183.

Baker, B.R., 1962, "Dynamic Stresses Created by a Moving Crack", Journal of Applied Mechanics, Vol. 29, p. 449.

Broberg, K.B., 1960, "The Propagation of a Brittle Crack", Arkiv For Fysik, Vol. 18, p. 159.

Craggs, J.W., 1960, "On the Propagation of a Crack in an Elastic Brittle Materials", Journal of the Mechanics and physics of Solids, Vol. 8, p. 66.

Eringen, A.C. and Suhubi, E.S., 1964, "Nonlinear Theory of Microelastic Solids", International Journal of Engineering Science, Vol. 2, p. 189, p. 389.

Eringen, A.C., 1966, "Linear Theory of Micropolar Elasticity", Journal of Mathematics and Mechanics, Vol. 15, No. 6, p. 909.

Eringen, A.C., Suhubi, 1974, "Elastodynamics", Vol. 2, Academic Press, New York.

Freund, L.B., 1972, "Crack Propagation in an Elastic Solid subjected to General Loading- Ⅱ.," Journal of the Mechanics and Physics of Solid, Vol. 20, p. 141.

Freund, L.B. and Clifton, R.J., 1974, "On the Uniqueness of Plane Elastodynamic Solutions for Running Cracks", Journal of Elasticity, Vol. 4, p. 293.

Gauthier, R.D. and Jahsman, W.E., 1979, "A Quest for Micropolar Elastic Constant- Ⅱ", Second Polish-Swedish Symposium in Microelastic Solids, Stockholm.

Han, S.Y., 1989, "Elastodynamic Analysis of a Propagating Finite Crack in a Micropolar Elastic Solid", Unpublished Ph. D. dissertation, Oregon State University, Corvallis, OR.

Kim, K.S., 1977, "Elastodynamic analysis of a propagating finite crack", Unpublished Ph. D. dissertation, University of Illinois at Urbana-Champaign, Urbana, IL.

Muskhelishvili, N.I., 1953, "Singular Intergral Equations", Noordhoff Publishing Co.

Sih, G.C. snd Loeber, J.F., 1969, "Wave Propagation in an Elastic Solid with a Line of Discontinuity or Finite Crack", Quarterly of Applied Mathematics, Vol. 27, p. 193.

Thau, S.A. and Lu, T.H., 1971, "Transient Stress Intensity Factors for a Finite Crack in an Elastic Solid Caused by a Dilatational Wave", International Journal of Solids and Structures, Vol. 7, p. 731.

Yoffe, E.H., 1951, "The Moving Griffith Crack", Philosophical Magagine, Vol. 42, p. 739.